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Convex Integration and Legendrian Approximation of Curves

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ABSTRACT. Using convex integration we give a constructive proof of the well-known fact that every continuous curve in a contact 3-manifold can be approximated by a Legendrian curve.

1. Introduction

A *contact structure* on a 3-manifold M is a maximally non-integrable rank 2 subbundle ξ of the tangent bundle of M. If α is a 1-form on M whose kernel is ξ, then ξ is a contact structure if and only if $\alpha \wedge \mathrm{d}\alpha \neq 0$. A curve η in a contact 3-manifold (M, ξ) is called *Legendrian*, whenever $\eta^*\alpha = 0$ for some (local) 1-form α defining ξ.

The purpose of this note is to give a detailed proof of the following statement which is often used in contact geometry and Legendrian knot theory.

Theorem 1.1. *Any continuous map from a compact* 1*-manifold to a contact* 3*-manifold can be approximated by a Legendrian curve in the* C^0*-Whitney topology.*

Whereas this theorem is a special case of Gromov's h-principle for Legendrian immersions [4], the curve-case can be treated by more elementary techniques. Sketches of proofs of Theorem 1.1 have already appeared in the literature, see for example [1, p.6-7], [2, p.40] or [3, p.102]. Exploiting the fact that every contact 3-manifold is locally contactomorphic to $\mathbb{R}^3$ equipped with the standard contact structure defined by $\alpha = \mathrm{d}z - y\mathrm{d}x$, Etnyre and Geiges indicate that either the *front-projection* (x, z) of a given curve (x, y, z) can be approximated by a zig-zag-curve whose slope approximates the y-component of the curve or the *Lagrangian projection* (x, y) can be approximated by a curve whose area integral approximates the z component of the curve, which can be achieved by adding small negatively or positively oriented loops.

Here, we give a different and analytically rigorous proof of Theorem 1.1 by using convex integration. Our proof has the advantage of providing a constructive approximation. In particular, in the case of a continuous curve in $\mathbb{R}^3$ equipped with the standard contact structure, we obtain an explicit Legendrian curve given in terms of an elementary integral. For instance, we obtain an explicit solution to the "parallel parking problem" in Example 3.1. Example 3.2 shows how our

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technique recovers the zig-zag-curves and the small loops in the front - respectively Lagrangian projections.

2. Proof of the Theorem

We start by first treating the case where the contact manifold is $\mathbb{R}^3$ equipped with the standard contact structure, that is, we aim to prove the following:

Proposition 2.1. *Let $\upsilon \in C^0([0,2\pi],\mathbb{R}^3)$. For every $\varepsilon > 0$ there exists a Legendrian curve $\eta \in C^\infty([0,2\pi],\mathbb{R}^3)$ such that $\|\upsilon - \eta\|_{C^0([0,2\pi])} \leqslant \varepsilon$.*

Remark 2.2. Here, as usual, $\|\gamma\|_{C^0(I)} := \sup_{t\in I}|\gamma(t)|$ and $\|\gamma\|_{C^1(I)} := \|\gamma\|_{C^0(I)} + \|\mathrm{d}\gamma\|_{C^0(I)}$.

Let the curve we wish to approximate be given by $(x,y,z) \in C^\infty([0,2\pi],\mathbb{R}^3)$. The regularity is no restriction due to a standard approximation argument using convolution. Let $\eta = (a,b,c) \in C^\infty([0,2\pi],\mathbb{R}^3)$ denote the approximating Legendrian curve. For every choice of smooth functions $(a,c) \in C^\infty([0,2\pi],\mathbb{R}^2)$ satisfying $\dot a \neq 0$, we obtain a Legendrian curve by defining $b = \dot c/\dot a$. Therefore, if $(\dot a(t), \dot c(t))$ lies in the set

$$\mathcal{R}_{t,\varepsilon} := \left\{(u,v) \in \mathbb{R}^2, |v - y(t)u| \leqslant \varepsilon \min\{|u|,|u|^2\}\right\},$$

for every $t \in [0,2\pi]$, then $\|b - y\|_{C^0([0,2\pi])} \leqslant \varepsilon$. This condition can be achieved by defining

$$(a(t),c(t)) := (x(0),z(0)) + \int_0^t \gamma(u,nu)\,\mathrm{d}u,$$

with $\gamma \in C^\infty([0,2\pi]\times S^1,\mathbb{R}^2)$ and $n \in \mathbb{N}$, provided that $\gamma(t,\cdot) \in \mathcal{R}_{t,\varepsilon}$. Furthermore, if γ additionally satisfies

$$\frac{1}{2\pi}\oint_{S^1} \gamma(t,s)\,\mathrm{d}s = (\dot x(t), \dot z(t)),$$

for all $t \in [0,2\pi]$, then – as we will show below – $(a(t),c(t))$ approaches $(x(t),z(t))$ as n gets sufficiently large.

The set $\mathcal{R}_{t,\varepsilon}$ is *ample*, i.e., the interior of its convex hull is all of $\mathbb{R}^2$. For any given point $(\dot x(t), \dot z(t)) \in \mathbb{R}^2$ we will thus be able to find a loop in $\mathcal{R}_{t,\varepsilon}$ having $(\dot x(t), \dot z(t))$ as its barycenter. This fact is sometimes referred to as the fundamental lemma of convex integration (see for instance [5, Prop. 2.11, p. 28]). In the particular case studied here we obtain an explicit formula for γ:

Lemma 2.3. *There exists a family of loops $\gamma \in C^\infty([0,2\pi]\times S^1,\mathbb{R}^2)$ satisfying $\gamma(t,\cdot) \in \mathcal{R}_{t,\varepsilon}$ and such that*

$$\frac{1}{2\pi}\oint_{S^1} \gamma(t,s)\,\mathrm{d}s = (\dot x(t), \dot z(t)), \tag{2.1}$$

for all $t \in [0,2\pi]$.

Proof. The map $\gamma := (\gamma_1,\gamma_2)$, where

$$\gamma_1(t,s) := r\cos s + \dot x(t)$$

and

$$\gamma_2(t,s) := \gamma_1(t,s)\left(y(t) + \frac{2(\dot z(t) - y(t)\dot x(t))}{r^2 + 2\dot x(t)^2}\gamma_1(t,s)\right)$$

satisfies (2.1) for every $r > 0$. If r is large enough one obtains $\gamma(t,\cdot) \in \mathcal{R}_{t,\varepsilon}$, where r can be chosen independently of t by compactness of $[0, 2\pi]$. □

We now have:

Proof of Proposition 2.1. With the definitions above we obtain

$$b(t) := \frac{\dot c(t)}{\dot a(t)} = y(t) + \frac{2(\dot z(t) - y(t)\dot x(t))}{r^2 + 2\dot x(t)^2}\gamma_1(t, nt).$$

We are left to show that $|(a,c) - (x,z)| \leqslant \varepsilon$ provided n is large enough. This follows from the following estimate

$$\|(a,c) - (x,z)\|_{C^0([0,2\pi])} \leqslant \frac{4\pi^2}{n}\|\gamma\|_{C^1([0,2\pi]\times S^1)}. \tag{2.2}$$

The estimate is in fact a geometric property of the derivative and can be interpreted as follows: Since $(\dot a, \dot c)$ and $(\dot x, \dot z)$ coincide "in average" on shorter and shorter intervals when n gets bigger and bigger, (a,c) and (x,z) tend to become close: Let

$$I_k := \left[\frac{2\pi k}{n}, \frac{2\pi(k+1)}{n}\right] \text{ for } k = 0, \dots, \left\lfloor \frac{nt}{2\pi} \right\rfloor - 1 \text{ and } J := \left[\left\lfloor \frac{nt}{2\pi}\right\rfloor \frac{2\pi}{n}, t\right].$$

Then we can estimate $D = |(a(t), c(t)) - (x(t), z(t))|$:

$$\begin{aligned}
D &= \left| \int_0^t \gamma(u, nu)\,\mathrm{d}u - \int_0^t (\dot x, \dot z)(u)\,\mathrm{d}u \right| \\
&\leqslant \sum_{k=0}^{\lfloor \frac{nt}{2\pi}\rfloor - 1} \left| \int_{I_k} \gamma(u, nu)\,\mathrm{d}u - \int_{I_k} \frac{1}{2\pi}\int_0^{2\pi} \gamma(u,v)\,\mathrm{d}v\,\mathrm{d}u \right| + \\
&\quad + \int_J \left(|\gamma(u,nu)| + \|\gamma\|_{C^0([0,2\pi]\times S^1)}\right)\,\mathrm{d}u \\
&\leqslant \sum_{k=0}^{\lfloor \frac{nt}{2\pi}\rfloor - 1} \left| \frac{1}{n}\int_0^{2\pi} \gamma\left(\frac{v + 2k\pi}{n}, v\right)\,\mathrm{d}v - \int_{I_k}\frac{1}{2\pi}\int_0^{2\pi}\gamma(u,v)\,\mathrm{d}v\,\mathrm{d}u \right| + \\
&\quad + \frac{4\pi}{n}\|\gamma\|_{C^0([0,2\pi]\times S^1)} \\
&\leqslant \sum_{k=0}^{\lfloor \frac{nt}{2\pi}\rfloor - 1} \left| \frac{1}{2\pi}\int_{I_k}\int_0^{2\pi}\left(\gamma\left(\frac{v+2k\pi}{n}, v\right) - \gamma(u,v)\right)\,\mathrm{d}v\,\mathrm{d}u \right| \\
&\quad + \frac{4\pi}{n}\|\gamma\|_{C^0([0,2\pi]\times S^1)} \\
&\leqslant \left\lfloor \frac{nt}{2\pi}\right\rfloor \frac{4\pi^2}{n^2}\|\partial_t\gamma\|_{C^0([0,2\pi]\times S^1)} + \frac{4\pi}{n}\|\gamma\|_{C^0([0,2\pi]\times S^1)} \\
&\leqslant \frac{4\pi}{n}\left(\pi\|\partial_t\gamma\|_{C^0([0,2\pi]\times S^1)} + \|\gamma\|_{C^0([0,2\pi]\times S^1)}\right).
\end{aligned}$$

By construction, the curve (a,b,c) is Legendrian and an approximation of (x,y,z), provided n is large enough. □

Next we show that we can approximate closed curves by closed Legendrian curves.

Proposition 2.4. *Let $\upsilon \in C^0(S^1, \mathbb{R}^3)$. For every $\varepsilon > 0$ there exists a Legendrian curve $\eta \in C^\infty(S^1, \mathbb{R}^3)$ such that $\|\upsilon - \eta\|_{C^0(S^1)} \leqslant \varepsilon$.*

Proof. Using standard regularization, let the curve we wish to approximate be given by $(x, y, z) \in C^\infty([0, 2\pi], \mathbb{R}^3)$, where the values of (x, y, z) in 0 and 2π agree to all orders. Define $g(t) := \gamma_1^2(t, nt)$. Since $\|g\|_{L^1([0,2\pi])} = O(r^2)$ as $r \to \infty$, we can choose $r > 0$ large enough such that $f := g/\|g\|_{L^1([0,2\pi])}$ is well-defined. With the notation

$$\mathrm{I}_2 := \int_0^{2\pi} \gamma_2(u, nu)\, \mathrm{d}u,$$

we define $\eta = (a, b, c)$ as follows:

$$(a(t), c(t)) := (x(0), z(0)) + \int_0^t \Big[\gamma(u, nu) - (0, \mathrm{I}_2 f(u))\Big]\, \mathrm{d}u, \tag{2.3}$$

and

$$b(t) := \frac{\dot c(t)}{\dot a(t)} = y(t) + \gamma_1(t, nt)\left(\frac{2(\dot z(t) - y(t)\dot x(t))}{r^2 + 2\dot x(t)^2} - \frac{\mathrm{I}_2}{\|g\|_{L^1([0,2\pi])}}\right). \tag{2.4}$$

A straightforward computation shows that the values of (a, b, c) in 0 and 2π agree to all orders, hence $\eta \in C^\infty(S^1, \mathbb{R}^3)$ and it is Legendre by construction. Using (2.2) we obtain $|\mathrm{I}_2| \leqslant \frac{4\pi^2}{n}\|\gamma_2\|_{C^1([0,2\pi]\times S^1)}$, hence we find using (2.4) as $r \to \infty$:

$$\|b - y\|_{C^0([0,2\pi])} \leqslant \|\gamma_1\|_{C^0([0,2\pi]\times S^1)}\left(1 + \frac{1}{n}\|\gamma\|_{C^1([0,2\pi]\times S^1)}\right) O(r^{-2}).$$

For the remaining components we find find using (2.2) and (2.3) the uniform bound

$$\begin{aligned}|(a(t), c(t)) - (x(t), z(t))| &\leqslant \frac{4\pi^2}{n}\|\gamma\|_{C^1([0,2\pi]\times S^1)} + \frac{|\mathrm{I}_2|}{\|g\|_{L^1([0,2\pi])}}\int_0^t g(u)\, \mathrm{d}u\\ &\leqslant \frac{8\pi^2}{n}\|\gamma\|_{C^1([0,2\pi]\times S^1)}.\end{aligned}$$

Choosing r large enough and $n \sim r^2$ concludes the proof. □

We show now how to glue together two local approximations of a curve Γ in M on two intersecting coordinate neighborhoods. Let therefore U_σ and U_τ in M be coordinate patches such that $U = U_\sigma \cap U_\tau \neq \emptyset$. Let I_σ and I_τ be compact intervals such that $I = I_\sigma \cap I_\tau$ contains an open neighborhood of $t = 0$ (after shifting the variable t if necessary) and such that $\Gamma(I_\sigma) \subset U_\sigma$, $\Gamma(I_\tau) \subset U_\tau$. Assume without restriction that Γ is smooth and let (x, y, z) represent Γ on U. Suppose that (x, y, z) is approximated by Legendrian curves $\sigma : I_\sigma \to \mathbb{R}^3$ and $\tau : I_\tau \to \mathbb{R}^3$ such that

$$\|\sigma - (x, y, z)\|_{C^0(I)} < \varepsilon^2, \quad \|\tau - (x, y, z)\|_{C^0(I)} < \varepsilon^2 \tag{2.5}$$

for some fixed $0 < \varepsilon < \frac{1}{2}$. For $r > 0$, define $R(r)$ to be the smallest number such that $\bar B_r(0) \subset \mathrm{conv}\left(\mathcal{R}_{0,\varepsilon} \cap \bar B_R(0)\right)$. Note that R depends continuously on r and if

$r > r_0 := \frac{\varepsilon}{\sqrt{1+y(0)^2}}$, then

$$R(r) = \frac{r}{\varepsilon}\sqrt{(1+y(0)^2)\left(1+(|y(0)|+\varepsilon)^2\right)} =: \frac{r}{\varepsilon}w(y(0),\varepsilon). \tag{2.6}$$

Choose $0 < \delta < \varepsilon^2$ such that $[-\delta,\delta] \subset I$ and such that $\delta\|(x,y,z)\|_{C^1(I)} \leqslant \varepsilon^2$ and define

$$\begin{aligned} p_1 &:= (\sigma_1(-\delta), \sigma_3(-\delta)),\\ \dot p_1 &:= (\dot\sigma_1(-\delta), \dot\sigma_3(-\delta)),\\ p_2 &:= (\tau_1(\delta), \tau_3(\delta)),\\ \dot p_2 &:= (\dot\tau_1(\delta), \dot\tau_3(\delta)). \end{aligned}$$

From (2.5) and the choice of δ we obtain $\dot p_1, \dot p_2 \in \mathscr{C}_\varepsilon := \left\{(u,v) \in \mathbb{R}^2, |v - y(0)u| \leqslant \varepsilon|u|\right\}$ and

$$\frac{p_2 - p_1}{2\delta} =: p \in B_{\bar r}(0), \text{where } \bar r = \frac{2\varepsilon^2}{\delta}.$$

Since $3\bar r > r_0$, we can express $R(3\bar r)$ by means of formula (2.6). This will be used in computation (2.9). We construct a path $\gamma = (\gamma_1,\gamma_2) : [-\delta,\delta] \to \mathscr{C}_\varepsilon$ as follows: For $\rho < \delta/2$, let $\gamma|_{[-\delta,-\delta+\rho]}$ be a continuous path from $\dot p_1$ to 0 and let $\gamma|_{[\delta-\rho,\delta]}$ be a continuous path from 0 to $\dot p_2$. We construct γ such that the quotient γ_2/γ_1 is well-defined on $[-\delta,-\delta+\rho] \cup [\delta-\rho,\delta]$ and equals $y(0)$ in $t = -\delta+\rho$ and $t = \delta - \rho$. Moreover, we require that

$$\int_{-\delta}^{-\delta+\rho} |\gamma(t)|\,\mathrm{d}t < \frac{\delta\varepsilon}{2} \text{ and } \int_{\delta-\rho}^{\delta} |\gamma(t)|\,\mathrm{d}t < \frac{\delta\varepsilon}{2}. \tag{2.7}$$

On $[-\delta,-\delta+\rho]$, such a path is for example given by

$$t \mapsto \left(1 - \frac{\delta+t}{\rho}\right)^k \begin{pmatrix} \dot\sigma_1(-\delta) \\ y(0)\dot\sigma_1(-\delta) + (\dot\sigma_3(-\delta) - y(0)\dot\sigma_1(-\delta))\left(1-\frac{\delta+t}{\rho}\right)^k \end{pmatrix}$$

provided $k \in \mathbb{N}$ is sufficiently large. We obtain

$$\frac{1}{2(\delta-\rho)}\left(2\delta p - \int_{-\delta}^{-\delta+\rho} \gamma(t)\,\mathrm{d}t - \int_{\delta-\rho}^{\delta}\gamma(t)\,\mathrm{d}t\right) =: \bar p \in B_{3\bar r}(0)$$

and hence $\bar p \in \operatorname{int}\operatorname{conv}(B_{R(3\bar r)}(0) \cap \mathscr{R}_{0,\varepsilon})$. Using the fundamental lemma of convex integration we let $\gamma|_{[-\delta+\rho,\delta-\rho]}$ be a continuous closed loop in $B_{R(3\bar r)}(0) \cap \mathscr{R}_{0,\varepsilon}$ based at 0 such that

$$\frac{1}{2(\delta-\rho)}\int_{-\delta+\rho}^{\delta-\rho} \gamma(t)\,\mathrm{d}t = \bar p.$$

With these definitions we obtain

$$\frac{1}{2\delta}\int_{-\delta}^{\delta}\gamma(t) = p.$$

Now we define $\eta = (a,b,c) : [-\delta,\delta] \to \mathbb{R}^3$ by letting $b(t) := \dot c(t)/\dot a(t)$, where

$$(a,c)(t) := p_1 + \int_{-\delta}^{t}\gamma(u)\mathrm{d}u.$$

The curve η is well-defined and Legendrian by construction. It satisfies $\eta(-\delta) = \sigma(-\delta)$ and $\eta(\delta) = \tau(\delta)$. Moreover, (a,c) and (σ_1,σ_3) agree to first order in $t = -\delta$

and so do (a,c) and (τ_1,τ_3) in $t=\delta$. From $\gamma([-\delta,\delta])\in\mathscr{C}_\varepsilon$ and the choice of δ we find

$$|b(t)-y(t)|\leqslant|b(t)-y(0)|+|y(t)-y(0)|\leqslant\varepsilon+\delta\|y\|_{C^1(I)}<2\varepsilon. \tag{2.8}$$

Using (2.5), (2.6), (2.7) and the choice of δ we obtain for the remaining components the uniform bound

(2.9)

$$\begin{aligned}|(a,c)(t)-(x,z)(t)|&\leqslant|p_1-(x,z)(-\delta)|+\int_{-\delta}^{t}(|\gamma(u)|+|(\dot x,\dot z)(u)|)\,\mathrm{d}u\\&\leqslant\varepsilon^2+\delta\varepsilon+\int_{-\delta+\rho}^{\delta-\rho}|\gamma(u)|\,\mathrm{d}u+2\delta\|(x,z)\|_{C^1(I)}\\&\leqslant2\varepsilon+2\delta R(3\bar r)\\&\leqslant\varepsilon\left(14+12\left(|y(0)|+\frac{1}{2}\right)^2\right).\end{aligned}$$

Finally, suppose υ is a continuous curve from a compact 1-manifold N (that is, N is a compact interval or S^1) into a contact 3-manifold (M,ξ). We fix some Riemannian metric g on M. Then it follows with the bounds (2.8),(2.9) and the compactness of the domain of υ that for every $\varepsilon>0$ there exists a ξ-Legendrian curve η such that

$$\sup_{t\in N}d_g(\upsilon(t),\eta(t))<\varepsilon,$$

where d_g denotes the metric on M induced by the Riemannian metric g. In particular, every open neighborhood of $\upsilon\in C^0(N,M)$ – equipped with the uniform topology – contains a Legendrian curve $N\to M$. Since N is assumed to be compact the uniform topology is the same as the Whitney C^0-topology, thus proving Theorem 1.1.

3. Examples

Example 3.1 (Parallel Parking). The trajectory of a car moving in the plane can be thought of as a curve $[0,2\pi]\to S^1\times\mathbb{R}^2$. Denoting by (φ,a,c) the natural coordinates on $S^1\times\mathbb{R}^2$, the angle coordinate φ denotes the orientation of the car with respect to the a-axis and the coordinates (a,c) the position of the car in the plane. Admissible motions of the car are curves satisfying

$$\dot a\sin\varphi=\dot c\cos\varphi.$$

The manifold $S^1\times\mathbb{R}^2$ together with the contact structure defined by the kernel of the 1-form $\theta:=\sin\varphi\,\mathrm{d}a-\cos\varphi\,\mathrm{d}c$ is a contact 3-manifold. Indeed, we have

$$\theta\wedge\mathrm{d}\theta=-\cos^2\varphi\,\mathrm{d}\varphi\wedge\mathrm{d}a\wedge\mathrm{d}c-\sin^2\varphi\,\mathrm{d}\varphi\wedge\mathrm{d}a\wedge\mathrm{d}c=-\mathrm{d}\varphi\wedge\mathrm{d}a\wedge\mathrm{d}c\neq0.$$

Applying Theorem 1.1 with $b=\tan\varphi$ gives an explicit approximation of the curve

$$t\mapsto(x(t),y(t),z(t))=(0,0,t).$$

Lemma 2.3 gives the loop

$$\gamma(t,s)=2(r\cos s,\cos^2 s),$$

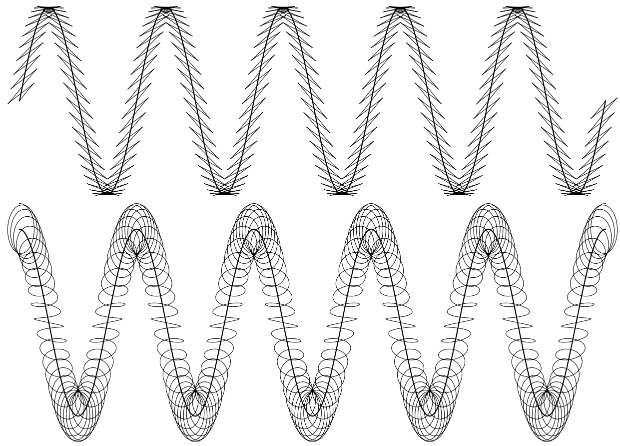

FIGURE 1. The front (top) and the Lagrangian projection (bottom) of the Legendrian approximation of η.

and hence the desired Legendrian curve

$$(\operatorname{arccot}(r\sec(nt)), 2rt\operatorname{sinc}(nt), t + t\operatorname{sinc}(2nt)),$$

provided r is large enough and $n \sim r^2$.

Example 3.2 (Legendrian Helix). The Legendrian approximation of the helix

$$\upsilon : [0, 2\pi] \to \mathbb{R}^3, \quad t \mapsto (t, \cos(5t), \sin(5t)),$$

with $n = \frac{2}{9}r^2$ and $r = 30$ is given by

$$\begin{aligned} a(t) &= t + \frac{3}{20}\sin(200t) \\ b(t) &= \frac{455}{451}\cos(5t) + \frac{120}{451}\cos(5t)\cos(200t) \\ c(t) &= \sin(5t) + \frac{459}{5863}\sin(195t) + \frac{1377}{18491}\sin(205t) + \frac{180}{35629}\sin(395t) + \\ &\quad + \frac{20}{4059}\sin(405t). \end{aligned}$$

and produces the zig-zags and the small loops in its front and Lagrangian projections (see Figure 1).

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